

Q.P. Code : 25318

(3 Hours)

[Total Marks : 80

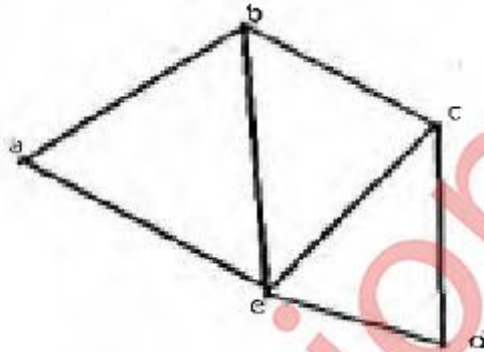
- N.B.: (1) Q1. is compulsory, attempt any 4 questions out of remaining six questions
(2) Assume any necessary data to justify the same
(3) Figures to the right indicate full marks
(4) Use of scientific calculator is allowed

Q1 a) Determine whether the relation on the set A is reflective, irreflexive, symmetric, asymmetric, antisymmetric or transitive. Give the necessary explanation to your answer. (10)
A=Set of all positive integers, aRb , iff $GCD(a,b)=1$

Q1 b) State the Tower of Hanoi problem and obtain the corresponding recurrence relation indicating the suitable initial conditions(s). Solve the recurrence relation obtained. (10)

Q2 a) Find the transitive closure of R of the following using by Warshall's algorithm
 $A=\{1,2,3,4,5\}$ $R=\{(1,1) (1,4) (2,2) (3,4) (3,5) (4,1) (5,2) (5,5)\}$ (10)

Q2 b) Find the adjacency list and adjacency matrix for the following graph (05)



Q3 a) Consider (3,6) encoding function e as follows: (10)
 $e(000) = 000000$, $e(001) = 000110$, $e(010) = 010010$, $e(011) = 010100$
 $e(100) = 100101$, $e(101) = 100011$, $e(110) = 110111$, $e(111) = 110001$
Show that the encoding function e is a group code.
Decode the code word 101101 with maximum likelihood technique.

Q3 b) Establish the following result without using truth tables. (use the laws of logic to show the following equivalence) (05)
 $(P \rightarrow Q) \wedge (R \rightarrow Q) \equiv (P \vee R) \rightarrow Q$.

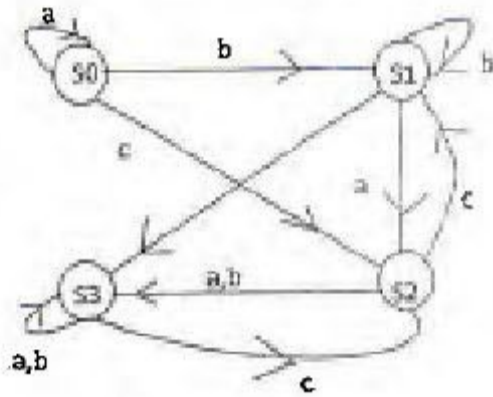
Q4 a) Let $V = \{v_0, w, a, b, c\}$ $S = \{a, b, c\}$ (10)
Let \rightarrow be the relation on V^* given by the relation
1. $V_0 \rightarrow aw$ 2. $w \rightarrow bbw$ 3. $w \rightarrow c$
Consider a phrase structure grammar $G = (V, S, v_0, \rightarrow)$
(i) Derive the sentence ab^4c . Also draw the derivation tree.
(ii) Derive the sentence ab^6c . Also draw the derivation tree.
(iii) Derive the sentence ab^8c . Also draw the derivation tree.

Q4 b) Find the solution of the recurrence relation defined by $a_n = 3a_{n-1} - 2a_{n-2}$ with $a_1 = 5$ (05)

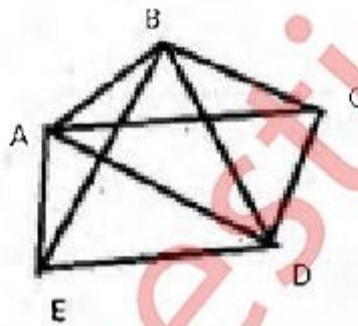
[TURN OVER

and $a_2=3$

- Q5 a) Let $A=\{1,2,3,4,12\}$. Consider the relation R as aRb iff 'a divides b' Show that R is a partial order relation. Draw the Hasse diagram of the Poset (A,R) . (10)
- Q5 b) Construct a transition table for a finite state machine whose diagram is shown below. (05)

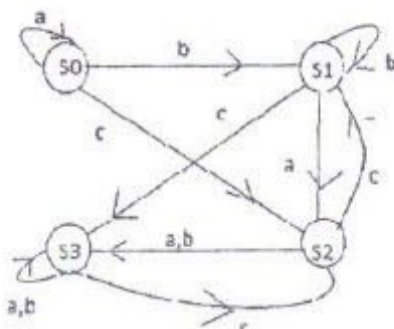


- Q6 a) Let S =Set of integers. Define the relation R on $A=S \times S$ as aRb if and only if $a \equiv b \pmod{2}$. (10)
- i) Show that R is an equivalence relation
- ii) Determine A/R .
- Q6 b) If G is a group with identity e . Show that if $a^2 = e$ for all a in G , then every element is its own inverse. (05)
- Q7 a) Consider the graph. Find an Euler path or Euler circuit, if exists. If it does not exist, why not? (10)



- Q7 b) Let T be the set of even integers. Show that $(\mathbb{Z}, +)$ and $(T, +)$ are isomorphic, where \mathbb{Z} is the set of integers. (05)

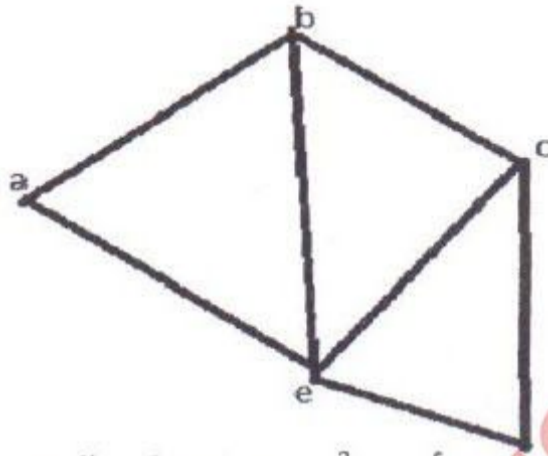
- Note (1) Q1. is compulsory, attempt any four questions out of remaining six questions.
 (2) Assume any necessary data but justify the same.
 (3) Figures to the right indicate full marks.
 (4) Answer to sub-questions should be grouped together.
- Q1. (a) Determine whether the relation R on set A is reflexive, irreflexive, symmetric, asymmetric, antisymmetric or transitive. 10
 A = set of all positive integers, aRb iff $GCD(a, b) = 1$
- (b) State the "Tower of Hanoi" problem and obtain the corresponding recurrence relation indicating the suitable initial conditions(s). Solve the recurrence relation obtained. 10
- Q2. (a) Determine whether the following set together with the binary operation is a semigroup, a monoid or neither. If it is a monoid, specify the identity. If it is a semigroup of a monoid determine whether it is commutative. 8
 Set $S = \{1, 2, 3, 6, 9, 18\}$ where $a * b = LCM. (a, b)$.
- (b) Use generating function method to solve recurrence relation 7
 $a_n = 3a_{n-1} + 2, a_0 = 2$
- Q3. (a) Obtain the PDNF and PCNF of the following: 8
 $(P \wedge Q) \vee (\sim P \wedge Q \wedge R)$
- (b) Determine the validity of the following argument:- 7
 If I go to my class tomorrow then I must get up early, and if I go to dance tonight I will stay up late. If I stay up late and get up early, then I will be forced to exist on only five hours of sleep. I simply cannot exist on only five hours of sleep. So I must either miss my class tomorrow or not go to the dance.
- Q4. (a) Let $A = \{1, 2, 3, 4, 12\}$ Consider the partial order relation R of divisibility on A. 8
 i.e. if $a, b \in A, aRb$ if and only if $a|b$. Draw the hasse diagram
- (b) Show that a group G is abelian if and only if $(a b)^2 = a^2 b^2$ the elements a & b are in G 7
- Q5. (a) Find particular solution of the recurrence relation $a_n = 7a_{n-1} - 10a_{n-2}$, with 8
 initial condition $a_0 = 1, a_1 = 8$
- (b) Construct a transition table for a finite state machine whose diagram is shown below. 7



[TURN OVER]

- Q6. (a) Find the adjacency list and adjacency matrix for the following graph

8



- (b) Show that the $(2,5)$ encoding function $e: B^2 \rightarrow B^5$ defined by

$$e(00)=00000 \quad e(10)=10101$$

$$e(01)=01110 \quad e(11)=11011$$

is a group code

7

- Q7. (a) Consider the group code defined by $e: B^2 \rightarrow B^5$ such that

$$e(00)=00000 \quad e(10)=10101$$

$$e(01)=01110 \quad e(11)=11011$$

Decode the following words relative to maximum likelihood decoding function (i) 11001 (ii) 10001

8

- (b) Let R be an equivalence relation on the set $A=\{1,2,3,4,5\}$ defined by $R=\{(1,1),(2,2),(3,3),(4,4),(5,5),(1,4),(4,1), (2,4),(4,2),(1,2),(2,1)\}$. Determine its equivalence classes of R & find A/R .

7

